Mark Scheme (Results)

Summer 2019

Pearson Edexcel GCE Further Mathematics
AS Further Pure 1 Paper 8FM0_21

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS
General Instructions for Marking

1. The total number of marks for the paper is 40.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses <u>and indicates which response they wish to submit</u>, examiners should mark this response.
If there are several attempts at a question <u>which have not been crossed out</u>, examiners should mark the final answer which is the answer that is the <u>most complete</u>.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

General Principles for Further Pure Mathematics Marking

(*But note that specific mark schemes may sometimes override these general principles*)

<u>Method mark for solving 3 term quadratic:</u>

1. Factorisation

$(x^2+bx+c)=(x+p)(x+q)$, where $|pq|=|c|$, leading to $x=\dots$

$(ax^2+bx+c)=(mx+p)(nx+q)$, where $|pq|=|c|$ and $|mn|=|a|$, leading to $x=\dots$

2. Formula

Attempt to use the correct formula (with values for *a*, *b* and *c*)

3. Completing the square

Solving $x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2\pm q\pm c=0,\ \ q\neq 0$, leading to $x=\dots$

<u>Method marks for differentiation and integration:</u>

1. Differentiation

Power of at least one term decreased by 1. $(x^n\to x^{n-1})$

2. Integration

Power of at least one term increased by 1. $(x^n\to x^{n+1})$

<u>Use of a formula</u>

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

<u>Method mark</u> for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is <u>not</u> quoted, the method mark can be gained by implication from <u>correct</u> working with values but may be lost if there is any mistake in the working.

<u>Exact answers</u>

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question	Scheme	Marks	AOs
1 (a)	$\{\sin x =\}\ \dfrac{2t}{1+t^2}$	B1	1.2
		(1)	
(b)(i)	$\left\{\tan\left(\dfrac{x}{2}\right)=\sqrt{2} \Rightarrow t=\sqrt{2} \Rightarrow\right\}\ \sin x = \dfrac{2(\sqrt{2})}{1+(\sqrt{2})^2}$ or $\dfrac{2(\sqrt{2})}{1+2}$	M1	1.1b
	$\sin x = \dfrac{2}{3}\sqrt{2}$ or $\dfrac{1}{3}\sqrt{8}$ or $\sqrt{\dfrac{8}{9}}$	A1	1.1b
		(2)	
(ii) Way 1	$\left\{\cos x \equiv \dfrac{\sin x}{\tan x} \Rightarrow\right\}\ \cos x = \dfrac{\frac{2t}{1+t^2}}{\frac{2t}{1-t^2}}$; $= \dfrac{1-t^2}{1+t^2}$ * cso	M1;	1.1b
		A1*	2.1
		(2)	
(ii) Way 2	$\left\{\tan x \equiv \dfrac{\sin x}{\cos x} \Rightarrow\right\}\ \dfrac{2t}{1-t^2} = \dfrac{\frac{2t}{1+t^2}}{\cos x}$; $\Rightarrow \cos x = \dfrac{1-t^2}{1+t^2}$ * cso	M1;	1.1b
		A1*	2.1
		(2)	
(ii) Way 3	$\{\sin^2 x + \cos^2 x \equiv 1 \Rightarrow\}\ \left(\dfrac{2t}{1+t^2}\right)^2 + \cos^2 x = 1$	M1	1.1b
	$\cos^2 x = 1 - \left(\dfrac{2t}{1+t^2}\right)^2 = \dfrac{(1+t^2)^2 - 4t^2}{(1+t^2)^2} = \dfrac{1-2t^2+t^4}{(1+t^2)^2} = \dfrac{(1-t^2)^2}{(1+t^2)^2}$ $\Rightarrow \cos x = \dfrac{1-t^2}{1+t^2}$ * cso	A1	2.1
		(2)	
(ii) Way 4	$\{o^2 + a^2 = h^2 \Rightarrow\}\ (2t)^2 + a^2 = (1+t^2)^2$	M1	1.1b
	$a^2 = (1+t^2)^2 - (2t)^2 = 1 - 2t^2 + t^4 = (1-t^2)^2$ $a = 1-t^2 \Rightarrow \cos x = \dfrac{1-t^2}{1+t^2}$ * cso	A1	2.1
		(2)	
(c)	$\{7\sin\theta + 9\cos\theta + 3 =\}\ 7\left(\dfrac{2t}{1+t^2}\right) + 9\left(\dfrac{1-t^2}{1+t^2}\right) + 3$	M1	1.1b
	$7\left(\dfrac{2t}{1+t^2}\right) + 9\left(\dfrac{1-t^2}{1+t^2}\right) + 3 = 0 \Rightarrow 14t + 9 - 9t^2 + 3 + 3t^2 = 0$ $\Rightarrow 6t^2 - 14t - 12 = 0 \Rightarrow 3t^2 - 7t - 6 = 0 \Rightarrow (t-3)(3t+2) = 0 \Rightarrow t = ...$	M1	1.1b
	Either $\left\{t = 3 \Rightarrow \dfrac{\theta}{2} = \arctan(3) \Rightarrow\right\}\ \theta = 2\arctan(3)$ **or** $\left\{t = -\dfrac{2}{3} \Rightarrow \dfrac{\theta}{2} = 180° + \arctan\left(-\dfrac{2}{3}\right) \Rightarrow\right\}\ \theta = 2\left(180° + \arctan\left(-\dfrac{2}{3}\right)\right)$	M1	1.1b
	$\dfrac{\theta}{2} = \{71.5650..., 146.3099...\} \Rightarrow \theta = \{143.1301..., 292.6198...\}$		
	$\theta = 143.1°, 292.6°$ (1dp)	A1	1.1b
		(4)	
			(9 marks)

Notes for Question 1	
(a)	
B1:	See scheme
(b)(i)	
M1:	Complete substitution of $t=\sqrt{2}$ into their expression from part (a)
A1:	Correct exact answer. See scheme.
Note:	Give M0 A0 for writing down the correct exact answer without any evidence of substituting $t=\sqrt{2}$ into $\sin x = \dfrac{2t}{1+t^2}$
Note:	For reference, $\sin x = \dfrac{2}{3}\sqrt{2} = 0.9428...$
(b)(ii)	**Way 1, Way 2 and Way 3**
M1:	Uses a correct trigonometric identity (or correct trigonometric identities) to find a correct expression which connects only $\cos x$ (or $\cos^2 x$) and t
A1*:	Correct proof
(b)(ii)	**Way 4**
M1:	Uses $\sin x = \dfrac{o}{h}$ and a correct Pythagoras method to express the adjacent edge of a triangle in terms of *t*.
A1*	Correct proof
(c)	
M1:	Uses at least one of $\sin\theta = \dfrac{2t}{1+t^2}$ or $\cos\theta = \dfrac{1-t^2}{1+t^2}$ to express $7\sin\theta + 9\cos\theta + 3$ in terms of *t* only
M1:	Uses both correct formula $\sin\theta = \dfrac{2t}{1+t^2}$ and $\cos\theta = \dfrac{1-t^2}{1+t^2}$ in $7\sin\theta + 9\cos\theta + 3 = 0$, multiplies both sides by $1+t^2$, forms a 3TQ and uses a correct method (e.g. using the quadratic formula, completing the square or a calculator approach) for solving their 3TQ to give $t = ...$
M1:	Uses both correct formula $\sin\theta = \dfrac{2t}{1+t^2}$ and $\cos\theta = \dfrac{1-t^2}{1+t^2}$ in $7\sin\theta + 9\cos\theta + 3 = 0$, adopts a correct *applied* strategy to find at least one value of θ within the range $0 < \theta \le 360°$ (or in radians $0 < \theta \le 2\pi$) such that either • $\theta = 2\arctan(\text{their found } t)$, where their found $t > 0$ • $\theta = 2(180° + \arctan(\text{their found } t))$, where their found $t < 0$ • $\theta = 2(180° - \arctan\lvert\text{their found } t\rvert)$, where their found $t < 0$
A1:	Correct answer only of $\theta = 143.1°,\ 292.6°$
Note:	Give A0 for extra solutions given within the range $0 < \theta \le 360°$
Note:	Ignore extra solutions outside the range $0 < \theta \le 360°$ for the A mark
Note:	Give 3rd M0 for $\dfrac{\theta}{2} = \{71.565..., 146.309...\}$ without attempting to find θ
Note	Give 3rd M0 for $\dfrac{\theta}{2} = \{71.565..., 146.309...\} \Rightarrow \theta = \{35.782..., 73.154...\}$
Note:	In degrees, $\dfrac{\theta}{2} = \{71.565...,\ 251.565...,\ -33.690...,\ 146.309...\}$
Note:	Working in radians gives $\dfrac{\theta}{2} = \{1.249..., 2.553...\} \Rightarrow \theta = \{2.498..., 5.107...\}$

Question	Scheme	Marks	AOs
2 (a)(i)	Line 3: Allow any of either • bracketing error • -24 should be 24 in the square brackets • $x(x+11)-x-24$ should be $x(x+11)-(x-24)$ • $x(x+11)-x-24$ should be $x(x+11)-x+24$	B1	2.3
(a)(ii)	Line 7: Allow any of either • should be $\{x \in \mathbb{R} : x < -12 \text{ or } -11 < x < 2 \text{ or } x > 24\}$ • they have found the regions where the inequality is < 0 • they have reversed the inequality	B1	2.3
		(2)	
(b) **Way 1**	$(x-24)(x+11)[x(x+11)-(x-24)] > 0$ $(x-24)(x+11)[x^2+10x+24] > 0$	M1	1.1b
	$(x-24)(x+11)(x+6)(x+4) > 0$ Critical values $x = -11, -6, -4, 24$	A1	1.1b
	$\{x \in \mathbb{R} : x < -11\} \cup \{x \in \mathbb{R} : -6 < x < -4\} \cup \{x \in \mathbb{R} : x > 24\}$	M1 A1	2.2a 2.5
		(4)	
(b) **Way 2**	$\frac{x}{x-24} > \frac{1}{x+11} \Rightarrow \frac{x}{x-24} - \frac{1}{x+11} > 0 \Rightarrow \frac{x(x+11)-(x-24)}{(x-24)(x+11)} > 0$	M1	1.1b
	$\Rightarrow \frac{x^2+10x+24}{(x-24)(x+11)} > 0 \Rightarrow \frac{(x+6)(x+4)}{(x-24)(x+11)} > 0$ Critical values $x = -11, -6, -4, 24$	A1	1.1b
	$\{x \in \mathbb{R} : x < -11\} \cup \{x \in \mathbb{R} : -6 < x < -4\} \cup \{x \in \mathbb{R} : x > 24\}$	M1 A1	2.2a 2.5
		(4)	
(b) **Way 3**	Considering $x < -11$ $\frac{x}{x-24} > \frac{1}{x+11} \Rightarrow x^2+11x > x-24 \Rightarrow x^2+10x+24 > 0$ gives $x < -6$ or $x > -4$. Hence $x < -11$	M1	1.1b
	Considering $-11 < x < 24$ $\frac{x}{x-24} > \frac{1}{x+11} \Rightarrow x^2+11x < x-24 \Rightarrow x^2+10x+24 < 0$ gives $-6 < x < -4$. Hence $-6 < x < -4$ Considering $x > 24$ $\frac{x}{x-24} > \frac{1}{x+11} \Rightarrow x^2+11x > x-24 \Rightarrow x^2+10x+24 > 0$ gives $x < -6$ or $x > -4$. Hence $x > 24$	A1	1.1b
	Overall, $\{x \in \mathbb{R} : x < -11\} \cup \{x \in \mathbb{R} : -6 < x < -4\} \cup \{x \in \mathbb{R} : x > 24\}$	M1 A1	2.2a 2.5
		(4)	
			(6 marks)

Notes for Question 2	
(a)(i)	
B1:	See scheme
Note:	Give B0 for contradictory reasons
(a)(ii)	**Way 1**
B1:	See scheme
Note:	Give B0 for contradictory reasons
Note:	Allow "Should be $x < -12$, $-11 < x < 2$, $x > 24$ "
Note:	Do not allow • "Should be $x < -12 \cap -11 < x < 2 \cap x > 24$ " • They have found where $x < 0$ and not where $x > 0$ • "There should be 3 inequalities and not 2 inequalities" • "The sign is the wrong way around"
(b)	**Way 1**
M1:	Uses brackets {to correct the error made on line 3}, forms a 3TQ and uses a correct method of solving a 3TQ to give $x = ...$
A1:	All four correct critical values for x
M1:	Deduces that the 2 "outsides" and the "middle interval" are required
A1:	Exactly 3 correct intervals. Their answer must be given in set notation. Accept equivalent set notation. E.g. Allow • $\{x \in \mathbb{R} : x < -11 \text{ or } -6 < x < -4 \text{ or } x > 24\}$ • $\{x < -11 \text{ or } -6 < x < -4 \text{ or } x > 24\}$ • $\{x < -11 \cup -6 < x < -4 \cup x > 24\}$ • $\mathbb{R} - ([-11, -6] \cup [-4, 24])$
Note:	Give final A0 for $\{x \in \mathbb{R} : x < -11\} \cap \{x \in \mathbb{R} : -6 < x < -4\} \cap \{x \in \mathbb{R} : x > 24\}$
Note:	Allow A1 for $\{x \in \mathbb{R} : x < -11, -6 < x < -4, x > 24\}$
(b)	**Way 2**
M1:	Gathers terms on one side and puts over a common denominator. Simplifies the numerator to $x(x+11) - (x-24)$ {and thereby corrects the error made in line 3}, forms a 3TQ and uses a correct method of solving a 3TQ to give $x = ...$
A1:	See Way 1
M1:	See Way 1
A1:	See Way 1
(b)	**Way 3**
M1:	Considers each of the intervals $x < -11$, $-11 < x < 24$, $x > 24$ separately and evaluates which parts (if any) of these regions satisfy the original inequality
A1:	Obtains a correct inequality statement for each of the intervals $x < -11$, $-11 < x < 24$, $x > 24$
M1:	See Way 1
A1:	See Way 1

Question	Scheme	Marks	AOs
3	{The population after 4 months is required over two iterations} $\Rightarrow h = \dfrac{1}{6}$	B1	3.3
	$\{ t_0 = 0,\ R_0 = 20 \Rightarrow \}\ \left(\dfrac{dR}{dt}\right)_0 = 2(20) + 4\sin 0\ \{= 40\}$	M1	3.4
	$\left\{\dfrac{R_1 - 20}{"(\frac{1}{6})"} = "40" \Rightarrow \right\}\ R_1 = 20 + "\tfrac{1}{6}""(40)"$	M1	1.1b
	$R_1 = \dfrac{80}{3}$ or awrt 26.7 or $20 + (\text{their } h)(40)$	A1ft	1.1b
	$\left(\dfrac{dR}{dt}\right)_1 = 2("R_1") + 4\sin("h") = 2\left(\dfrac{80}{3}\right) + 4\sin\left(\dfrac{1}{6}\right)\ \{= 53.9969...\}$	M1	1.1b
	$R_2 = R_1 + h\left(\dfrac{dR}{dt}\right)_1 = \dfrac{80}{3} + \dfrac{1}{6}(53.9969...) = 35.666... = 35$ or 36 rabbits	A1	1.1b
	$R_2 = 35.666... \approx 35$ or $36 < 40$ Julie will not be able to start to sell her rabbits after 4 months.	B1ft	3.2a
		(7)	
			(7 marks)

Notes for Question 3

B1:	Translates the situation given to state (or use) the correct value for the step length h
M1:	Uses the model to find the initial value of $\dfrac{dR}{dt}$ using the initial condition $t_0 = 0,\ R_0 = 20$
M1:	Applies the approximation formula with $R_0 = 20$, their stated h, their $\left(\dfrac{dR}{dt}\right)_0$ to find a numerical expression for R_1
A1:	**depends on both previous M marks** At 2 months, finds the approximation for R as $\dfrac{80}{3}$ or awrt 26.7
Note:	Only give the following follow through. i.e. Allow A1ft for $20 + (\text{their } h)(40)$ for their stated h
M1:	Attempts to find a numerical expression for $\left(\dfrac{dR}{dt}\right)_1$ with their $\dfrac{80}{3}$ and $t_1 =$ their h
A1:	Applies the approximation formula for a second time to give R_2 as a truncated 35 or a value in the interval $[35.5, 36]$
B1ft:	Attempts two iterations of their $R_{n+1} = R_n + h\left(\dfrac{dR}{dt}\right)_n$ to find a value for R_2. Compares their value of R_2 with 40 (which can be implied) and draws a conclusion about whether Julie will be able to start to sell her rabbits after 4 months.
Note:	Give final B0 for applying more than or fewer than two iterations before comparing
Note:	Using $h = \dfrac{1}{12}$ yields $R_1 = 23.3333...,\ R_2 = 27.2499...,\ R_3 = 31.8469...,\ R_4 = 37.2372...$
Note:	Give special case final A1 for giving R_4 as a truncated 37 or a value in the interval $[37, 37.4]$
Note:	Therefore, using $h = \dfrac{1}{12}$ with four iterations can gain a maximum B0 M1 M1 A1 M1 A1 B0
Note:	Answers in the range $[35.5, 36]$ can follow from an incorrect method. E.g. Give final M0 A0 for using $h = \dfrac{1}{6}$, $\left(\dfrac{dR}{dt}\right)_1 = 2\left(\dfrac{80}{3}\right) + 4\sin(\underline{0.1}) = 53.73266... \Rightarrow R_2 = \dfrac{80}{3} + \dfrac{1}{6}(53.73266...) = 35.622...$

Question	Scheme	Marks	AOs
4	$A(2, 1, 4)$, $B(6, 1, 2)$, $C(4, 10, 3)$, $D(5, 8, d)$		
(a) Way 1	Uses appropriate vectors in a correct method to make a complete attempt to find the area of triangle ABC.	M1	3.1b
	$\overrightarrow{AB} = \begin{pmatrix} 4 \\ 0 \\ -2 \end{pmatrix}$, $\overrightarrow{AC} = \begin{pmatrix} 2 \\ 9 \\ -1 \end{pmatrix}$, $\left\{\overrightarrow{BC} = \begin{pmatrix} -2 \\ 9 \\ 1 \end{pmatrix}\right\}$	M1	1.1b
	e.g. $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 0 & -2 \\ 2 & 9 & -1 \end{vmatrix} = \dots$ or $\begin{pmatrix} 4 \\ 0 \\ -2 \end{pmatrix} \times \begin{pmatrix} 2 \\ 9 \\ -1 \end{pmatrix} = \dots$	M1	1.1b
	$= 18\mathbf{i} + 0\mathbf{j} + 36\mathbf{k}$		
	Area $ABC = \dfrac{1}{2}\sqrt{(18)^2 + (0)^2 + (36)^2}$		
	$\{= 20.1246...\} = 9\sqrt{5}\ (\text{cm}^2)$ or awrt $20.1\ (\text{cm}^2)$	A1	2.2a
		(4)	
(a) Way 2	Uses appropriate vectors to find an angle or perpendicular height in triangle ABC and uses a correct method to make a complete attempt to find the area of triangle ABC.	M1	3.1b
	$\overrightarrow{AB} = \begin{pmatrix} 4 \\ 0 \\ -2 \end{pmatrix}$, $\overrightarrow{AC} = \begin{pmatrix} 2 \\ 9 \\ -1 \end{pmatrix}$, $\left\{\overrightarrow{BC} = \begin{pmatrix} -2 \\ 9 \\ 1 \end{pmatrix}\right\}$	M1	1.1b
	Uses a correct method to find an angle or perpendicular height in triangle ABC	M1	1.1b
	Note: $B\hat{A}C = 27.905...$, $A\hat{B}C = 76.047...$, $B\hat{C}A = 76.047...$ or perpendicular height $= 9$		
	Area $ABC = \dfrac{1}{2}\sqrt{86}\sqrt{20}\sin 76.047...$ or $\dfrac{1}{2}\sqrt{86}\sqrt{86}\sin 27.905...$ or $\dfrac{1}{2}\sqrt{20}(9)$		
	$\{= 20.1246...\} = 9\sqrt{5}\ (\text{cm}^2)$ or awrt $20.1\ (\text{cm}^2)$	A1	2.2a
		(4)	
(b)	Finds appropriate vectors to form the equation volume tetrahedron $ABCD = 21$ to give a linear equation in d **Note:** The volume must include $\dfrac{1}{6}$	M1	3.1a
	e.g. $\left\| \begin{pmatrix} 3 \\ 7 \\ d-4 \end{pmatrix} \bullet \begin{pmatrix} 18 \\ 0 \\ 36 \end{pmatrix} \right\| = \dots$ or $\begin{vmatrix} 4 & 0 & -2 \\ 2 & 9 & -1 \\ 3 & 7 & d-4 \end{vmatrix} = \dots$	M1	1.1b
	$= \|54 + 36d - 144\|$ or $\|4(9d - 36 + 7) - 2(14 - 27)\|$ $\{= \|36d - 90\|\}$	A1	1.1b
	$\left\{\dfrac{1}{6}\|36d - 90\| = 21 \Rightarrow \|36d - 90\| = 126 \Rightarrow\right\}$ $d = 6$	A1	1.1b
		(4)	
			(8 marks)

Question	Scheme	Marks	AOs
4	$A(2, 1, 4),\ B(6, 1, 2),\ C(4, 10, 3),\ D(5, 8, d)$		
(a) Way 3	Complete attempt to find the area of triangle ABC by applying $\frac{1}{2}\left\lvert \overrightarrow{OA}\times\overrightarrow{OB} + \overrightarrow{OB}\times\overrightarrow{OC} + \overrightarrow{OC}\times\overrightarrow{OA}\right\rvert$ or equivalent	M1	3.1b
	$\overrightarrow{OA}\times\overrightarrow{OB} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 1 & 4 \\ 6 & 1 & 2 \end{vmatrix} = \ldots$ **and** $\overrightarrow{OB}\times\overrightarrow{OC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6 & 1 & 2 \\ 4 & 10 & 3 \end{vmatrix} = \ldots$, **and** $\overrightarrow{OC}\times\overrightarrow{OA} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 10 & 3 \\ 2 & 1 & 4 \end{vmatrix} = \ldots$	M1	1.1b
	$\{\overrightarrow{OA}\times\overrightarrow{OB} + \overrightarrow{OB}\times\overrightarrow{OC} + \overrightarrow{OC}\times\overrightarrow{OA}\} = \begin{pmatrix} -2 \\ 20 \\ -4 \end{pmatrix} + \begin{pmatrix} -17 \\ -10 \\ 56 \end{pmatrix} + \begin{pmatrix} 37 \\ -10 \\ -16 \end{pmatrix}$	M1	1.1b
	Area $ABC = \frac{1}{2}\sqrt{(18)^2 + (0)^2 + (36)^2}$		
	$\{= 20.1246...\} = 9\sqrt{5}\ (\text{cm}^2)$ or awrt $20.1\ (\text{cm}^2)$	A1	2.2a
		(4)	

Notes for Question 4

(a)	**Way 1**
M1:	Complete correct process of taking the vector product between 2 edges of triangle *ABC*, applying Pythagoras and multiplying the result by 0.5
M1:	Uses a correct method to find any 2 edges of triangle *ABC*
M1:	Attempts to take the vector cross product between 2 edges of triangle *ABC*
A1:	Deduces the correct area of either $9\sqrt{5}\ (\text{cm}^2)$ or awrt $20.1\ (\text{cm}^2)$
(a)	**Way 2**
M1:	See scheme
M1:	Uses a correct method to find any 2 edges of triangle *ABC*
M1:	Either • finds an angle in *ABC* by using a correct scalar product method • finds an angle in *ABC* by using the cosine rule in the correct direction • realises triangle *ABC* is isosceles and applies Pythagoras in the correct direction to find the perpendicular height
A1:	Deduces the correct area as either $9\sqrt{5}\ (\text{cm}^2)$ or awrt $20.1\ (\text{cm}^2)$
Note:	For Way 1 and Way 2, using any of $\overrightarrow{OA}$, $\overrightarrow{OB}$ or $\overrightarrow{OC}$ in their vector product is M0 M0 A0 A0
(a)	**Way 3**
M1:	See scheme
M1:	Attempts to apply $\overrightarrow{OA}\times\overrightarrow{OB}$, $\overrightarrow{OB}\times\overrightarrow{OC}$ and $\overrightarrow{OC}\times\overrightarrow{OA}$
A1:	Attempts to add (as vectors) the results of applying $\overrightarrow{OA}\times\overrightarrow{OB}$, $\overrightarrow{OB}\times\overrightarrow{OC}$ and $\overrightarrow{OC}\times\overrightarrow{OA}$
A1:	Deduces the correct area as either $9\sqrt{5}\ (\text{cm}^2)$ or awrt $20.1\ (\text{cm}^2)$

Notes for Question 4 Continued	
(b)	
M1:	See scheme
M1:	Uses appropriate vectors in an attempt at the scalar triple product
A1:	Correct applied expression for the scalar triple product (allow $\pm$ and ignore modulus sign)
A1:	Correct solution leading to $d=6$
Note:	Using any of $\overrightarrow{OA}, \overrightarrow{OB}, \overrightarrow{OC}$ or $\overrightarrow{OD}$ in their scalar triple product is M0 M0 A0 A0
Note:	Some vector product calculations for reference:
	$\left\lvert\overrightarrow{AD}.\left(\overrightarrow{AB}\times\overrightarrow{AC}\right)\right\rvert = \begin{vmatrix} 3 & 7 & d-4 \\ 4 & 0 & -2 \\ 2 & 9 & -1 \end{vmatrix} = \left\lvert \begin{pmatrix} 3 \\ 7 \\ d-4 \end{pmatrix} \bullet \begin{pmatrix} 18 \\ 0 \\ 36 \end{pmatrix} \right\rvert = \lvert 54+36d-144\rvert = \lvert 36d-90\rvert$
	$\left\lvert\overrightarrow{AB}.\left(\overrightarrow{AC}\times\overrightarrow{AD}\right)\right\rvert = \begin{vmatrix} 4 & 0 & -2 \\ 2 & 9 & -1 \\ 3 & 7 & d-4 \end{vmatrix} = \left\lvert \begin{pmatrix} 4 \\ 0 \\ -2 \end{pmatrix} \bullet \begin{pmatrix} 9d-29 \\ 5-2d \\ -13 \end{pmatrix} \right\rvert = \lvert 36d-116+26\rvert = \lvert 36d-90\rvert$
	$\left\lvert\overrightarrow{AC}.\left(\overrightarrow{AB}\times\overrightarrow{AD}\right)\right\rvert = \begin{vmatrix} 2 & 9 & -1 \\ 4 & 0 & -2 \\ 3 & 7 & d-4 \end{vmatrix} = \left\lvert \begin{pmatrix} 2 \\ 9 \\ -1 \end{pmatrix} \bullet \begin{pmatrix} 14 \\ 10-4d \\ 28 \end{pmatrix} \right\rvert = \lvert 28+90-36d-28\rvert = \lvert 90-36d\rvert$

Question	Scheme	Marks	AOs
5	$H: xy = c^2,\ c > 0;\ P\left(ct, \frac{c}{t}\right)$ lies on H; $OB = 2OA$; Area$(OAB) = 32$		
Way 1	**Either** $y = \frac{c^2}{x} = c^2 x^{-1} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = -c^2 x^{-2}$ or $-\frac{c^2}{x^2}$ **or** $xy = c^2 \Rightarrow x\frac{\mathrm{d}y}{\mathrm{d}x} + y = 0$ **or** $x = cp,\ y = \frac{c}{p} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}p}.\frac{\mathrm{d}p}{\mathrm{d}x} = -\left(\frac{c}{p^2}\right)\left(\frac{1}{c}\right)$; condone $t \equiv p$ **and** so, at $P\left(ct, \frac{c}{t}\right),\ m_T = -\frac{1}{t^2}$	M1	3.1a
	$y - \frac{c}{t} = "-\frac{1}{t^2}"(x - ct)$ **or** $\frac{c}{t} = "-\frac{1}{t^2}"(ct) + b \Rightarrow y = "-\frac{1}{t^2}"x +$ their $b \Rightarrow y = -\frac{1}{t^2}x + \frac{2c}{t}$	M1 A1	1.1b 1.1b
	$y = 0 \Rightarrow x = 2ct\ \{\Rightarrow x_A = 2ct\},\ \ x = 0 \Rightarrow y = \frac{2c}{t}\ \left\{\Rightarrow y_B = \frac{2c}{t}\right\}$	M1 A1	1.1b 1.1b
	$\{OB = 2OA \Rightarrow\}\ \frac{2c}{t} = 2(2ct) \Rightarrow t = ...$	M1	2.1
	$\left\{t^2 = \frac{1}{2} \Rightarrow\right\}\ t = \frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$ or awrt 0.707	A1	1.1b
	$\{\text{Area } (OAB) = 32 \Rightarrow\}\ \frac{1}{2}(2ct)\left(\frac{2c}{t}\right) = 32 \Rightarrow c = ...\ \{\Rightarrow c = 4\}$	M1	2.1
	Deduces the ***numerical*** value x_P and y_P using their values of t and c	M1	2.2a
	$P(2\sqrt{2}, 4\sqrt{2})$ **or** P(awrt 2.83, awrt 5.66) **or** $x = 2\sqrt{2}$ **and** $y = 4\sqrt{2}$	A1	1.1b
		(10)	
Way 2	Same requirement as the 1st M mark in Way 1	M1	3.1a
	e.g. $\left\{t = \frac{1}{\sqrt{2}} \Rightarrow P\left(\frac{c}{\sqrt{2}}, \sqrt{2}c\right) \Rightarrow\right\}\ y - \sqrt{2}c = -2\left(x - \frac{c}{\sqrt{2}}\right)$ using $m_T = -2$ and their P which has been found by a correct method	M1 A1	1.1b 1.1b
	$y = 0 \Rightarrow x = \sqrt{2}c\ \{\Rightarrow x_A = \sqrt{2}c\},\ \ x = 0 \Rightarrow y = 2\sqrt{2}c\ \{\Rightarrow y_B = 2\sqrt{2}c\}$	M1 A1	1.1b 1.1b
	$\{OB = 2OA \Rightarrow\}\ m_T = -2$ and their $m_T = -\frac{1}{t^2} = -2 \Rightarrow t = ...$	M1	2.1
	$\left\{t^2 = \frac{1}{2} \Rightarrow\right\}\ t = \frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$ or awrt 0.707 $\left\{\Rightarrow P\left(\frac{c}{\sqrt{2}}, \sqrt{2}c\right)\right\}$	A1	1.1b
	$\{\text{Area } (OAB) = 32 \Rightarrow\}\ \frac{1}{2}\sqrt{2}c\left(2\sqrt{2}c\right) = 32 \Rightarrow c = ...\ \{\Rightarrow c = 4\}$	M1	2.1
	Deduces the ***numerical*** value x_P and y_P using their values of t and c	M1	2.2a
	$P(2\sqrt{2}, 4\sqrt{2})$ **or** P(awrt 2.83, awrt 5.66) **or** $x = 2\sqrt{2}$ **and** $y = 4\sqrt{2}$	A1	1.1b
		(10)	
		(10 marks)	

Question	Scheme	Marks	AOs
5	$H: xy = c^2,\ c > 0;\ P\left(ct, \frac{c}{t}\right)$ lies on H; $OB = 2OA$; Area$(OAB) = 32$		
Way 3	Same requirement as the 1st M mark in Way 1	M1	3.1a
	e.g. $y - 8\sqrt{2} = -2(x - 0)$ or $y - 0 = -2(x - 4\sqrt{2})$	M1	1.1b
	using $m_T = -2$ and either their $A(4\sqrt{2}, 0)$ or their $B(0, 8\sqrt{2})$ which have been found by a correct method	A1	1.1b
	$\{$Area $(OAB) = 32,\ OB = 2OA \Rightarrow\}\ \frac{1}{2}(x)(2x) = 32 \Rightarrow x = ...$	M1	2.1
	$x = 4\sqrt{2}\ \{\Rightarrow x_A = 4\sqrt{2}\}$ or $y = 8\sqrt{2}\ \{\Rightarrow y_B = 8\sqrt{2}\}$	A1	1.1b
	$\{OB = 2OA \Rightarrow\}\ m_T = -2$ and their $m_T = -\frac{1}{t^2} = -2 \Rightarrow t = ...$	M1	2.1
	$\left\{t^2 = \frac{1}{2} \Rightarrow\right\}\ t = \frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$ or awrt 0.707 $\left\{\Rightarrow P\left(\frac{c}{\sqrt{2}}, \sqrt{2}c\right)\right\}$	A1	1.1b
	$\sqrt{2}c - 8\sqrt{2} = -2\left(\frac{c}{\sqrt{2}} - 0\right) \Rightarrow c = ...\ \{\Rightarrow c = 4\}$	M1	1.1b
	Deduces the ***numerical*** value x_P and y_P using their values of t and c	M1	2.2a
	$P(2\sqrt{2}, 4\sqrt{2})$ **or** P(awrt 2.83, awrt 5.66) **or** $x = 2\sqrt{2}$ **and** $y = 4\sqrt{2}$	A1	1.1b
		(10)	
Way 4	Complete process substituting their $y - 8\sqrt{2} = -2(x - 0)$ or $y - 0 = -2(x - 4\sqrt{2})$ **into** $xy = c^2$ and applying $b^2 - 4ac = 0$ to their resulting $2x^2 - 8\sqrt{2}x + c^2 = 0$	M1	3.1a
	e.g. $y - 8\sqrt{2} = -2(x - 0)$ or $y - 0 = -2(x - 4\sqrt{2})$	M1	1.1b
	using $m_T = -2$ and either their $A(4\sqrt{2}, 0)$ or their $B(0, 8\sqrt{2})$ which have been found by a correct method	A1	1.1b
	$\{$Area $(OAB) = 32,\ OB = 2OA \Rightarrow\}\ \frac{1}{2}(x)(2x) = 32 \Rightarrow x = ...$	M1	2.1
	$x = 4\sqrt{2}\ \{\Rightarrow x_A = 4\sqrt{2}\}$ or $y = 8\sqrt{2}\ \{\Rightarrow y_B = 8\sqrt{2}\}$	A1	1.1b
	dependent on 2nd M mark $\{xy = c^2 \Rightarrow\}\ x(-2x + 8\sqrt{2}) = c^2\ \{\Rightarrow 2x^2 - 8\sqrt{2}x + c^2 = 0\}$	dM1	2.1
	or $\{xy = c^2 \Rightarrow\}\ \frac{1}{2}\left(8\sqrt{2} - y\right)y = c^2\ \{\Rightarrow y^2 - 8\sqrt{2}y + 2c^2 = 0\}$	A1	1.1b
	$\{b^2 - 4ac = 0 \Rightarrow\}\ (8\sqrt{2})^2 - 4(2)(c^2) = 0 \Rightarrow c = ...\ \{\Rightarrow c = 4\}$	M1	1.1b
	Deduces the ***numerical*** value x_P and y_P using their value of c	M1	2.2a
	$P(2\sqrt{2}, 4\sqrt{2})$ **or** P(awrt 2.83, awrt 5.66) **or** $x = 2\sqrt{2}$ **and** $y = 4\sqrt{2}$	A1	1.1b
		(10)	
Note:	**<u>For the final M1 mark in Way 1, Way 2, Way 3 and Way 4</u>** Allow final M1 for a correct method which gives any of $x_P = 2\sqrt{2}$ **or** $y_P = 4\sqrt{2}$ **or** $x_P =$ awrt 2.83 **or** $y_P =$ awrt 5.66 o.e.		

Notes for Question 5	
Way 1	
M1:	Establishes the gradient of the tangent by differentiating $xy = c^2$ • to give $\frac{\mathrm{d}y}{\mathrm{d}x} = \pm k\,x^{-2}$; $k \neq 0$, or • by the product rule to give $\pm x\frac{\mathrm{d}y}{\mathrm{d}x} \pm y$, or • by parametric differentiation to give $\left(\text{their } \frac{\mathrm{d}y}{\mathrm{d}t}\right) \times \frac{1}{\left(\text{their } \frac{\mathrm{d}x}{\mathrm{d}t}\right)}$, condoning $p \equiv t$ **and** attempt to use $P\left(ct, \frac{c}{t}\right)$ to write down the gradient of the tangent to the curve in terms of t
M1:	Correct straight line method for an equation of a tangent where $m_T\ (\neq m_N)$ is found by using calculus. **Note:** m_T must be a function of t for this mark
A1:	Correct equation of the tangent which can be simplified or un-simplified
M1:	Attempts to find either the x-coordinate of A or the y-coordinate of B
A1:	**Both** {x-coordinate of A is} $2ct$ **and** the {y-coordinate of B is} $\frac{2c}{t}$
M1:	See scheme
A1:	See scheme
M1:	See scheme
M1:	See scheme
A1:	See scheme
Way 2	
M1:	Same description as the 1st M mark in Way 1
M1:	See scheme
A1:	Correct equation of the tangent which can be simplified or un-simplified
M1:	Attempts to find either the x-coordinate of A or the y-coordinate of B
A1:	**Both** {x-coordinate of A is} $\sqrt{2}c$ **and** the {y-coordinate of B is} $2\sqrt{2}c$
M1:	Recognising that the gradient of the tangent is -2 and puts this equal to their $\frac{\mathrm{d}y}{\mathrm{d}x}$ and finds $t = \ldots$
A1:	See scheme
M1:	See scheme
M1:	See scheme
A1:	See scheme
Way 3	
M1:	Same description as the 1st M mark in Way 1
M1:	See scheme
A1:	Correct equation of the tangent which can be simplified or un-simplified
M1:	Uses $y = 2x$ and Area $(OAB) = 32$ to find either x_A or y_B
A1:	Either {x-coordinate of A is} $4\sqrt{2}$ or the {y-coordinate of B is} $8\sqrt{2}$
M1:	Recognising that the gradient of the tangent is -2 and puts this equal to their $\frac{\mathrm{d}y}{\mathrm{d}x}$ and finds $t = \ldots$
A1:	See scheme
M1:	Substitutes their P (which is in terms of c, and has come from a correct method) into the equation of the tangent and finds $c = \ldots$
M1:	See scheme
A1:	See scheme

Notes for Question 5	
Way 4	
M1:	See scheme
M1:	See scheme
A1:	Correct equation of the tangent which can be simplified or un-simplified
M1:	Uses $y = 2x$ and Area $(OAB) = 32$ to find either x_A or y_B
A1:	Either {x-coordinate of A is} $4\sqrt{2}$ or the {y-coordinate of B is} $8\sqrt{2}$
M1:	See scheme
A1:	See scheme
M1:	See scheme
M1:	See scheme
A1:	See scheme

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